Q.P. Code – 42139

First Semester B.Sc. Degree Examination, October/November 2018

(CBCS – Semester Scheme)

Mathematics

Paper 1.1 — ALGEBRA AND CALCULUS – I

Time : 3 Hours] *[Max. Marks : 90*

Instructions to Candidates :

1. *Answer ALL questions.*
2. *Answers should be written completely in English.*

PART – A

I. Answer any **SIX** questions : **(6 × 2 = 12)**

1. Find the nth derivative of $\log(4x^2 - 9)$.

2. Find the angle between radius vector and the tangent for the curve $r = a\cos\theta$.

3. If $Y = b\log\cos(x/b)$ then show that $\frac{dS}{dX} = \sec\left(\frac{x}{b}\right)$.

4. Evaluate : $\int_0^{\pi/2} \cos^8 x \, dx$

5. If $u = ax^2 + 2hxy + by^2$ show that $xU_x + yU_y = 2u$.

6. If $u = x(1-y)$ and $V = xy$ then find $\frac{\partial(u,v)}{\partial(x,y)}$.

7. Find the rank of the matrix $A = \begin{bmatrix} -2 & -1 & 3 & -1 \\ 1 & 2 & 3 & -1 \\ 1 & 0 & 1 & 1 \end{bmatrix}$.

8. If 'λ' is an Eigen value of the matrix A, then prove that 'λ^2' is an Eigen value of A^2.

PART – B

II. Answer any **SIX** questions : **(6 × 3 = 18)**

1. Find $D^n[\cos^3 x]$.

2. If $p = r\sin\phi$ then prove that $\frac{1}{p^2} = \frac{1}{r^2} + \frac{1}{r^4}\left(\frac{dr}{d\theta}\right)^2$.

3. Find the radius of curvature for the curve

   $Y = 4\sin x - \sin 2x$ at $x = \frac{\pi}{2}$.

4. Evaluate : $\int_0^{\pi} x\sin^4 x\, dx$

5. If $u = x^2 + xy + y^2$ and $y = \sin x$, find $\frac{du}{dx}$.

6. If $u = \log\left(\frac{x^2 + y^2}{x + y}\right)$ prove that $xU_x + yU_y = 1$.

7. Find the value of '*a*' in order that the matrix $A = \begin{bmatrix} 2 & 4 & -4 & a \\ -1 & -2 & -1 & 2 \\ 1 & 2 & -1 & 3 \end{bmatrix}$ has rank 2.

8. If $\lambda = 4$ and $\lambda = -1$ are the Eigen values of the matrix $\begin{bmatrix} 1 & 2 \\ 3 & 2 \end{bmatrix}$, find corresponding Eigen vector.

PART – C

III. Answer any **THREE** questions : **(3 × 5 = 15)**

1. Find the *n*th derivative of $e^{ax}\sin(bx + c)$.

2. If $Y = e^{a\sin^{-1}x}$ prove that $(1 - x^2)y_{n+2} - (2n + 1)x\,y_{n+1} - (n^2 + a^2)y_n = 0$.

3. Prove that the Pedal equation of the cardioid $r = a(1+\cos\theta)$ is $2ap^2 = r^3$.

4. With usual Notation prove that

$$\cot\phi = \frac{1}{r}\frac{dr}{d\theta}$$

IV. Answer any **THREE** questions : **(3 × 5 = 15)**

1. With usual Notation prove that $\rho = \frac{[(\dot{x})^2 + (\dot{y})^2]^{3/2}}{\dot{x}\ddot{y} - \dot{y}\ddot{x}}$.

2. Show that the evolute of the curve $x = a[\cos\theta + \theta\sin\theta]$; $y = a[\sin\theta - \theta\cos\theta]$ is $x^2 + y^2 = a^2$.

3. Find all asymptotes of the curve

$$y^3 - x^2y + 2y^2 + 4y + 1 = 0$$

4. Obtain the reduction formula for $\int \cos^n x\, dx$.

V. Answer any **THREE** questions : **(3 × 5 = 15)**

1. State the prove Euler's Theorem.

2. Expand $e^x \cos y$ using Taylor's Expansion at $\left(1, \frac{\pi}{4}\right)$ upto second degree.

3. If $u = \frac{yz}{x}$, $v = \frac{zx}{y}$, $w = \frac{xy}{z}$

Then show that $\frac{\partial(u, v, w)}{\partial(x, y, z)} = 4$ at (1, 1, 1)

4. Find the Extreme value of $f(x, y) = x^3 + y^3 - 3xy$.

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VI. Answer any **THREE** questions : **(3 × 5 = 15)**

1. Solve :

$$\begin{aligned} x + 2y + 3z &= 0 \\ y + 5z &= 0 \\ 3x + 2y + z &= 0 \\ 2x + 3z &= 0 \end{aligned}$$

2. For what value of "λ" the system of equations

$$\begin{aligned} x + 2y + 3z &= 14 \\ x + 4y + 7z &= 30 \\ x + y + z &= \lambda \end{aligned}$$

is consistent and hence solve.

3. Diagonalize the matrix $\begin{pmatrix} 3 & 4 \\ -2 & -3 \end{pmatrix}$.

4. Find the adjoint of the matrix $\begin{pmatrix} 1 & 2 \\ 3 & 2 \end{pmatrix}$ using Cayley-Hamilton theorem.

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First Semester B.Sc. Degree Examination, October/November 2018

(Semester Scheme)

Mathematics

Paper I — MATHEMATICS

Time : 3 Hours] *[Max. Marks : 90*

Instructions to Candidates :

1. *Answer ALL questions.*
2. *Answers should be written completely in English.*

I. Answer any **FIFTEEN** of the following : **(15 × 2 = 30)**

1. Define a truth set.

2. Negate : $(\forall x)\{\sim p(x) \Rightarrow q(x)\}$.

3. Define partition of a set. Give an example.

4. Let R is R' be two relations defined on A. Show that if R and R' are reflexive, then $R \cup R'$ is Reflexive.

5. Find the second order derivative of $y = \log(\cos x)$.

6. Write the nth derivative of $\log(ax+b)$.

7. If $y = x^n$ the find y_{n+1}.

8. If $x = r\cos\theta$ and $y = r\sin\theta$, show that $\left(\frac{\partial r}{\partial x}\right)^2 + \left(\frac{\partial r}{\partial y}\right)^2 = 1$.

9. Define homogeneous function. Give an example.

10. Find $\frac{dy}{dx}$ for $x^3 + y^3 - 3axy = 0$, using partial differentiation.

11. Evaluate : $\int_{-\pi/2}^{\pi/2} \sin^5 x \, dx$

12. Evaluate : $\int_0^{\pi/2} \sin^6 x \cos^8 x \, dx$

13. Find the direction cosines of the line whose direction ratios are 6, 2, 3.

14. Using the direction ratios show that the lines AB and CD are parallel, where $A = (1, 2, 3)$, $B(4, -3, 6)$, $C = (-1, 2, -2)$ and $D = (2, -3, 1)$.

15. Write the condition for the lines with direction cosines l_1, m_1, n_1 and l_2, m_2, n_2 to be

(a) parallel

(b) perpendicular

16. Write the condition for coplanarity of four points.

17. Find the angle between the planes

$3x - 6y + 2z + 5 = 0$ and $4x - 12y + 3z - 3 = 0$

18. Find the center and radius of the sphere

$x^2 + y^2 + z^2 + 2x - 4y - 6z + 5 = 0$

19. Find the equation of a right circular cone with vertex $(1, -1, 0)$, the axis $\frac{x-1}{1} = \frac{y+1}{3} = \frac{z-0}{2}$ and semi vertical angle is $60°$.

20. Write the equation of right circular cylinder in Cartesian form.

II. Answer any **TWO** of the following : **(2 × 5 = 10)**

1. Prove that $T[p(x) \wedge q(x)] = T[p(x)] \cap T[q(x)]$

2. Negate, "All odd numbers are not prime numbers and some prime numbers are even".

3. Let R be the relation defined on the set z of integers by $R = \{(x, y) / x - y$ is divisible by $5\}$. Show that R is an Equivalence Relation.

4. Prove that the composition of bijective functions is a bijection.

III. Answer any **THREE** of the following : **(3 × 5 = 15)**

1. Find the nth derivative of $e^{ax}\sin(bx+c)$.

2. If $y = (\sin^{-1} x)^2$, show that

$(1-x^2)y_{n+2} - (2n+1)y_{n+1} - n^2 y_n = 0$

3. If $u = f(r)$ where $r^2 = x^2 + y^2$, show that $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = f''(r) + \frac{1}{r} f'(r)$

4. State and prove Euler's theorem for a homogeneous function of two variables.

5. If $x = r\cos\theta$, $y = r\sin\theta$, find $J = \frac{\partial(x, y)}{\partial(r, \theta)}$ and $J' = \frac{\partial(r, \theta)}{\partial(x, y)}$. Also verify $J \cdot J' = 1$.

IV. Answer any **TWO** of the following : **(2 × 5 = 10)**

1. Obtain the Reduction formula for $\int \tan^n x\, dx$.

2. Evaluate : $\int_0^4 x^3\sqrt{4x - x^2}\; dx$

3. Using Leibnitz's rule of differentiation under integral sign. Evaluate $\int_0^1 \frac{x^a - 1}{\log x}\, dx$, where '$a$' is a parameter.

V. Answer any **THREE** of the following : **(3 × 5 = 15)**

1. If α, β, γ are the angles made by a line with the co-ordinate axes, prove that

(a) $\sin^2\alpha + \sin^2\beta + \sin^2\gamma = 2$

(b) $1 + \cos 2\alpha + \cos 2\beta + \cos 2\gamma = 0$

2. Find the angle between the two lines whose direction ratios are 1, –1, 2 and 1, 0, –3.

3. Show that the points (0, –1, 0), (2, 1, –1), (1, 1, 1) and (3, 3, 0) are coplanar. Find the equation of the plane passing through them.

4. Find the image of the point (–3, 0, 1) in the plane $4x - 3y + 2z = 19$.

5. Find the length and the equations of the shortest distance between the lines $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and $\frac{x-2}{3} = \frac{y-6}{4} = \frac{z-5}{5}$.

VI. Answer any **TWO** of the following : **(2 × 5 = 10)**

1. Find the equation of a sphere passing through the points (1, 0, 0), (0, 1, 0), (0, 0, 1) and (2, –1, 1). Find its centre and radius.

2. Find the equation of the right circular cone whose vertex is at (2, –3, 5), axis makes equal angles with the co-ordinate axes and the semi vertical angle is measured to be 30°.

3. Find the equation of the right circular cylinder of radius 2 and whose axis is $\frac{x-1}{2} = \frac{y+3}{-1} = \frac{z-3}{5}$.

Q.P. Code – 42139

First Semester B.Sc. Degree Examination, October/November 2019

(CBCS Scheme)

Mathematics

Paper 1.1 - ALGEBRA AND CALCULUS – I

Time : 3 Hours] *[Max. Marks : 90*

Instructions to Candidates :

1. *Answer ALL Questions.*
2. *Answer should be written completely in English.*

PART – A

Answer any **SIX** of the following : **(6 × 2 = 12)**

1. Find the nth derivative of $\cos^2 x$.

2. Define the point of inflexion.

3. For the curve $x = a\cos t$ and $y = b\sin t$. Find $\frac{ds}{dt}$.

4. Evaluate $\int_0^{\pi/2} \cos^8 x dx$.

5. If $u = \log(x^2 + y^2 + z^2)$ find $\frac{\partial u}{\partial x}$ and $\frac{\partial u}{\partial y}$.

6. If $u = 2x - 3y$ and $v = 5x + 4y$, find the Jacobian of (u, v) w.r.t (x, y).

7. Find the rank of the matrix $A = \begin{bmatrix} 1 & 2 & -1 & 3 \\ 2 & 4 & -4 & 7 \\ -1 & -2 & -1 & 2 \end{bmatrix}$

8. Prove that eigen vector of a matrix corresponds to one and only one eigen value of A.

PART – B

Answer any **SIX** of the following : **(6 × 3 = 18)**

9. Find the nth derivative of $\frac{1}{x^2+3x-10}$.

10. Find the length of perpendicular from pole to the tangent for the curve $r = a(1+\cos\theta)$.

11. Find $\frac{ds}{dx}$ for the curve $x = a(1-\cos\theta)$ and $y = a(\theta+\sin\theta)$.

12. Evaluate $\int_0^{\pi} x\sin^5 x\,dx$.

13. If $u = \tan^{-1}(y/x)$ prove that $\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=0$.

14. If $u = \sin^{-1}(x-y)$, $x = 3t$ and $y = 4t^3$ then show that $\frac{du}{dt}=\frac{3}{\sqrt{1-t^2}}$.

15. Find the value of 'a' using row reduced echelon form where $A=\begin{bmatrix} 6 & a & -1 \\ 2 & 3 & 1 \\ 3 & 4 & 2 \end{bmatrix}$ has rank 2.

16. Find the eigen values of the matrix $A=\begin{bmatrix} 3 & 2 & -2 \\ -3 & -1 & 3 \\ 1 & 2 & 0 \end{bmatrix}$.

PART– C

Answer any **THREE** of the following : **(3 × 5 = 15)**

17. Find the nth derivative of $e^{ax}\cdot\cos(bx+c)$.

18. If $y = e^{m\sin^{-1}x}$ prove that $(1-x^2)y_{n+2}-(2n+1)xy_{n+1}-(n^2+m^2)y_n=0$.

19. With usual notations prove that $\tan\phi = r\dfrac{d\theta}{dr}$ for the curve $r = f(\theta)$.

20. Find the pedal equation of the curve $x^2 + y^2 = 2ax$.

PART– D

Answer any **THREE** of the following : **(3 × 5 = 15)**

21. Find the radius of curvature at any point for the curve $x = a\cos^3 t$, $y = a\sin^3 t$.

22. Show that the evaluate of the curve $x = a(\cos t + t\sin t)$ and $y = a(\sin t - t\cos t)$ is $x^2 + y^2 = a^2$.

23. Find all the asymptotes of the curve $x^3 + x^2 y - xy^2 - y^3 - 3x - y - 1 = 0$.

24. Obtain the reduction formula for $\int_0^{\pi/2} \cos^n x dx$.

PART – E

Answer any **THREE** of the following : **(3 × 5 = 15)**

25. State and prove Euler's theorem on homogeneous function.

26. If $u = x^2 - 2y$, $v = x + y$, then find $J = \dfrac{\partial(u,v)}{\partial(x,y)}$ and $J' = \dfrac{\partial(x,y)}{\partial(u,v)}$ also verify that $J \cdot J' = 1$.

27. Expand $e^x \cos y$ using Taylor's theorem at $(1, \pi/4)$ upto second degree terms.

28. Find the extreme value of $f(x,y) = x^3 + 3xy^2 - 15x^2 - 15y^2 + 72x$.

PART – F

Answer any **THREE** of the following : **(3 × 5 = 15)**

29. Solve completely the system of equations

$$x_1 + 2x_2 + 2x_3 = 0$$
$$2x_1 + x_2 + x_3 = 0$$
$$3x_1 + 2x_2 + 2x_3 = 0$$
$$x_2 + x_3 = 0$$

30. For what values of M and n the system of equations

$$x+2y+3z=4$$
$$x+3y+4z=5 \text{ has}$$
$$x+3y+mz=n$$

(a) unique solution

(b) no solution

(c) infinite number of solutions.

31. Diagonalize the matrix $\begin{bmatrix} 7 & 3 \\ 3 & -1 \end{bmatrix}$.

32. Verify the Cayley-Hamilton theorem for the matrix $A=\begin{bmatrix} 1 & 1 \\ 2 & 1 \end{bmatrix}$ and hence find A^{-1}.

Q.P. Code – 42139

First Semester B.Sc. Degree Examination, November 2017

(CBCS – Semester Scheme)

Mathematics

Paper 1.1 – ALGEBRA AND CALCULUS – I

Time : 3 Hours] *[Max. Marks : 90*

Instructions to Candidates :

1) *Answer ALL questions.*

2) *Answers should be written completely in English.*

PART – A

I. Answer any **SIX** questions : **(6 × 2 = 12)**

1. Find the nth derivative of $\sin(ax+b)$.

2. Show that for the curve $r = a\theta$, the polar subtangent varies as the square of the radius vector.

3. Find the radius of curvature at any point (p, r) on the curve $r^3 = a^2 p$.

4. Evaluate $\int_0^{\pi/2} \sin^3 x \cdot \cos^2 x \, dx$

5. If $u = x\sin y + y\sin x$ then find $\frac{\partial u}{\partial x}, \frac{\partial u}{\partial y}$.

6. If $u = 2x - 3y$, $v = 5x + 4y$ then find $\frac{\partial(u, v)}{\partial(x, y)}$.

7. Find the rank of the matrix $A = \begin{bmatrix} 1 & 2 & -1 & 4 \\ 2 & 4 & 3 & 5 \\ 3 & 2 & 6 & 7 \end{bmatrix}$.

8. If λ is an eigen value of the matrix A, then prove that λ^2 is an eigen value of A^2.

PART – B

II. Answer any **SIX** questions : **(6 × 3 = 18)**

1. Find the nth derivative of $\frac{1}{(x+2)(x-1)}$.

2. Find the length of perpendicular from pole to the tangent to the curve $r = a(1-\cos\theta)$.

3. Find the centre of curvature for $xy = 12$ at (3, 4).

4. Evaluate $\int_0^{\pi} \cos^4 x \, dx$.

5. Find $\frac{du}{dt}$, if $u = e^x \sin y$ where $x = \log t$, $y = t^2$.

6. If $x^3 + y^3 - 3axy = 0$, then find $\frac{dy}{dx}$ using partial differentiation.

7. Find the value of a in order that the matrix $A = \begin{bmatrix} 2 & 4 & -4 & a \\ -1 & -2 & -1 & 2 \\ 1 & 2 & -1 & 3 \end{bmatrix}$ has rank 2.

8. Find the eigen values and eigen vectors of the matrix $\begin{bmatrix} 4 & 1 \\ -1 & 2 \end{bmatrix}$.

PART – C

III. Answer any **THREE** questions : **(3 × 5 = 15)**

1. If $y = a\cos(\log x) + b\sin(\log x)$ then show that

   $x^2 y_{n+2} + (2n+1)xy_{n+1} + (n^2+1)y_n = 0$.

2. Find the points of inflexion on the curve $x = \log\left(\frac{y}{x}\right)$.

3. With usual notations, prove that $\tan\phi = r\frac{d\theta}{dr}$ for the curve $r = f(\theta)$.

4. Show that the pedal equation of the parabola $y^2 = 4a(x+a)$ is $p^2 = ar$.

IV. Answer any **THREE** questions : **(3 × 5 = 15)**

1. Find the radius of curvature at any point for the curve $x = a\cos^3 t$, $y = a\sin^3 t$.

2. Find the envelope of the family of lines $x\cos^3\alpha + y\sin^3\alpha = a$, where '$\alpha$' is a parameter.

3. Obtain the reduction formula for $\int \tan^n x\, dx$ and hence find $\int_0^{\pi/4} \tan^n x\, dx$.

4. Show that $\int_0^{\pi} \frac{\log(1 + a\cos x)}{\cos x}\, dx = \pi \sin^{-1}(a)$ where 'a' is the parameter and $-1 < a < 1$ by using differentiation under the integral sign.

V. Answer any **THREE** questions : **(3 × 5 = 15)**

1. State and prove Euler's theorem for a homogeneous function of two variables.

2. If $u = \sin^{-1}\left(\frac{x^2 + y^2}{x + y}\right)$, then show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \tan u$.

3. If $x = r\cos\theta$, $y = r\sin\theta$ then find $J = \frac{\partial(x, y)}{\partial(r, \theta)}$ and $J' = \frac{\partial(r, \theta)}{\partial(x, y)}$. Also verify $J \cdot J' = 1$.

4. Expand $e^x \cos y$ in a Taylor's expansion about the point $(1, \pi/4)$ upto second degree terms.

VI. Answer any **THREE** questions : **(3 × 5 = 15)**

1. Solve completely the system of equations

$$\begin{aligned} x + 3y + 2z &= 0 \\ 2x - y + 3z &= 0 \\ 3x - 5y + 4z &= 0 \\ \text{and } x + 17y + 4z &= 0. \end{aligned}$$

2. For what values of μ and η the system $x + 3y + 4z = 5$, $x + 2y + z = 3$, $x + 3y + \mu z = \eta$ has (a) no solution (b) unique solution and (c) many solutions.

3. Diagonalize the matrix $\begin{bmatrix} 5 & 4 \\ 4 & -1 \end{bmatrix}$.

4. Verify the Cayley-Hamilton theorem for the matrix $A = \begin{bmatrix} 4 & -1 \\ 1 & 2 \end{bmatrix}$ and hence find A^{-1}.

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I Semester B.Sc. Examination, Nov./Dec. 2013
(Semester Scheme)
MATHEMATICS (Paper – I)

Time : 3 Hours — Max. Marks : 90

***Instructions**: 1) Answer **all** questions.*
*2) Answer should be written in **English** only.*

I. Answer **any fifteen** of the following : **(15×2=30)**

1) Write the rules of negating a quantified open sentence.

2) Find the truth set of the open sentence. P (x) : x is a prime divisor of 36 where R [P(x)] = Z.

3) Define partition of a set. Give an example.

4) Show that $f : z \to 2z$ where z is the set of integers and 2z is the set of all even integers, defined by f(x) = 2x is a bijective mapping.

5) Find the n^{th} derivative of $\sin^3 x$.

6) Find the n^{th} derivative of $xe^{2x}\cos 3x$.

7) If $Z = x^y$, verify $\frac{\partial^2 Z}{\partial x \partial y} = \frac{\partial^2 Z}{\partial y \partial x}$

8) If $U = x^4 \log\left(\frac{y}{x}\right)$, then show that $x\frac{\partial U}{\partial x} + y\frac{\partial U}{\partial y} = 4U$

9) If $U = x^2 + y^2$, where $x = t$, $y = t^2$ find $\frac{dU}{dt}$ using total derivative.

10) If $x = U^2 - V^2$ and $y = 2UV$. Show that $\frac{\partial (x, y)}{\partial (U, V)} = 4(u^2 + v^2)$

11) Evaluate $\int_{-\pi/2}^{\pi/2} \sin^8 x \, dx$

12) Evaluate $\int_0^{\pi/4} \tan^5 x - dx$

13) Find the coordinates of the point which divides the line join of (2, –3, 1) and (3, 2, –1) in the ratio 2 : 3.

14) A line makes angles 45° and 60° with the positive x and y axes respectively. Find the angle made by the line with the z-axis.

15) Write the condition for the lines with direction. Cosines l_1, m_1, n_1, and l_2, m_2, n_2 to be

i) parallel

ii) perpendicular

16) Find the equation of the plane through the points (1, 1, 0), (1, 2, 1) and (–2, 2, –1).

17) Show that the planes 2x – 4y + 3z + 5 = 0 and 10x + 11y + 8z – 7 = 0 are perpendicular.

18) Find the centre and radius of the sphere $x^2 + y^2 + z^2 + 2x - 4y - 6z + 5 = 0$.

19) Write the equation of the right circular cylinder in Cartesian form.

20) Find the equation of the right circular cone whose vertex is the origin whose semivertical angle is 45° and axis along z-axis.

II. Answer **any two** of the following : **(2×5=10)**

1) Show that $T[p(x) \vee q(x)] = T[p(x)] \cup T[q(x)]$.

2) Prove the following statement by indirect proof : "If x + y is even then x and y are either both odd or both even".

3) Let R be the relation defined on the set Z of integers by $R = \{(x, y) \mid x - y \text{ is divisible by } 7\}$. Show that R is an equivalence relation.

4) Define inverse of a function. If $f : A \to B$ is a bijection, then prove that the inverse function $f^{-1} : B \to A$ is also a bijection.

III. Answer **any three** of the following : **(3×5=15)**

1) Find the n^{th} derivative of $e^{ax} \sin(bx + c)$.

2) If $\cos^{-1}\left(\frac{y}{b}\right) = \log\left(\frac{x}{n}\right)^n$, then show that $x^2 y_{n+2} + (2n + 1) x.y_{n+1} + 2n^2 y_n = 0$.

3) If $V = \text{Tan}^{-1}\left(\frac{x^3 + y^3}{x - y}\right)$, then show that $x\frac{\partial V}{\partial x} + y\frac{\partial V}{\partial y} = \text{Sin}2V$.

4) If U = f (x – y, y – z, z – x) then show that $\frac{\partial U}{\partial x} + \frac{\partial U}{\partial y} + \frac{\partial U}{\partial z} = 0$.

5) If $x = r\cos\theta$, $y = r\sin\theta$ then show that $JJ^1 = 1$.

IV. Answer **any two** of the following : **(2×5=10)**

1) Obtain the reduction formula for $\int \sin^n x dx$ and hence evaluate $\int_0^{\pi/2} \text{Sin}^n x \, . \, dx$

2) Evaluate $\int_0^{\infty} \frac{x^3}{(4 + x^2)^{5/2}} dx$

3) Evaluate $\int_0^{\infty} e^{-x^2} \cos\alpha x \, dx$, where α is a parameter, using Leibnitz rule given that $\int_0^{\infty} e^{-x^2} dx = \frac{\sqrt{\pi}}{2}$

V. Answer **any three** of the following : **(3×5=15)**

1) Find the direction cosines of two lines which are connected by the relation $l - 5m + 3n = 0$ and $7l^2 + 5m^2 - 3n^2 = 0$.

2) Derive vector equation of a line passing through the points where position vectors are $\vec{a}$ and $\vec{b}$. Deduce its Cartesian form.

3) Find the equation of the plane passing through (1, 1, 1) and perpendicular to the planes $\vec{r}.(1, -3, 5) + 1 = 0$ and $\vec{r}.(3, -1, 7) = 3$.

4) Find the image of the point (2, 3, –1) in the plane 3x – y + 2z + 12 = 0.

5) Find k, such that the lines $\frac{x-3}{1} = \frac{y-2}{3} = \frac{z-1}{4}$ and $\frac{x-4}{2} = \frac{y-2}{3} = \frac{z+2}{k}$ are coplanar and find the plane containing them.

VI. Answer **any two** of the following : **(2×5=10)**

1) Find the vector equation of the sphere whose centre is at the point whose position vector is $3\hat{i} - \hat{j} + 4\hat{k}$ and passing through the point whose position vector is $\hat{i} - 2\hat{j}$. Write its Cartesian form.

2) Find the equation of the right circular cone whose axis is $\frac{x-1}{-1} = \frac{y-2}{3} = \frac{z-3}{3}$ and a generator is $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$.

3) Find the equation of the right circular cylinder, whose radius is 2 units and axis passes through the points (1, –3, 2) and (–1, –2, –3).

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I Semester B.Sc. Examination, November/December 2014
(Semester Scheme)
MATHEMATICS (Paper – I)

Time : 3 Hours Max. Marks : 90

Instructions *: 1) Answer **all** questions.*
*2) Answer should be written in **English** only.*

I. Answer **any fifteen** questions. **(15×2=30)**

1) Name two kinds of quantifiers, write their notations.

2) Find the truth set of the open sentence $P(x) = x^3 - 1 = 0$ where $R[P(x)]$ = set of all complex numbers.

3) Define an equivalence relation.

4) If $f : R \to R$, $g : R \to R$ are defined by $f(x) = 2x + 1$, $g(x) = 5 - 3x$ find $g \circ f$.

5) Find the n^{th} derivative of $x^2 e^{3x}$.

6) Find the n^{th} derivative of $\cos^3 x$.

7) If $U = x^6 + y^6$ show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 6U$.

8) If $U = x \sin y + y \sin x$ find $\frac{\partial^2 u}{\partial y \partial x}$.

9) If $U = e^y \sin x$ where $x = t^2$, $y = \log t$ find $\frac{\partial u}{\partial t}$.

10) If $U = x+y$ and $v = xy$ find $\frac{\partial(u,v)}{\partial(x,y)}$.

P.T.O.

11) Evaluate $\int_{0}^{\pi/2} \cos^6 x dx$.

12) Evaluate $\int_{-\pi/2}^{\pi/2} \sin^4 x . \cos^6 x \, dx$.

13) If the direction ratio's of a line are (2, 3,– 6). Find its direction cosines.

14) Show that the lines whose direction ratios are (2, 3, 4) and (1, – 2, 1) are at right angles.

15) Find the length of the perpendicular from the point (1, –1, 3) on to the plane $5x + 2y - 7z + 9 = 0$.

16) Find the equation of the plane passing through (3, – 3, 1) and parallel to the plane $2x + 3y + 5z + 6 = 0$.

17) Find the angle between the line $\frac{x-1}{2} = \frac{y+1}{1} = \frac{z-2}{1}$ and the plane $x - y + z + 1 = 0$.

18) Find the equation of the sphere described on the line joining the points (2, 4, 4) and (2, –2, 2) as a diameter.

19) Find the equation of right circular cone with vertex at the origin, semi-vertical angle 30° and axis along y-axis.

20) Write the equation of the right circular cylinder in the Cartesian form.

II. Answer **any two** of the following : **(2×5=10)**

1) With usual notations prove that $T[p(x) \wedge q(x)] = T[p(x)] \cap T[q(x)]$.

2) If p(x): x is a divisor of 100 and q(x) : $x^2 < 40$ find the truth set of $p(x) \wedge q(x)$ and $\sim q(x)$ in z.

3) If $f : A \rightarrow B$ and $g : B \rightarrow C$ are two bisective mapping then prove that $(g \circ f)^{-1} = f^{-1} \circ g^{-1}$.

4) The mappings $f : Q \rightarrow Q$ is defined by $f(x) = 3x + 5 \ \forall \ x \in Q$ show that f is a bisective and find f^{-1}.

III. Answer **any three** of the following. **(3×5=15)**

1) Find the n^{th} derivative of $\frac{x-1}{(x-2)^2(x+2)}$.

2) If $x = \sin t$ and $y = \cos pt$ show that $(1-x^2)y_{n+2} - (2n+1)xy_{n+1} - (n^2 - p^2)y_n = 0$.

3) If $U = \frac{1}{\sqrt{x^2+y^2+z^2}}$ show that $\frac{\partial^2 U}{\partial x^2} + \frac{\partial^2 U}{\partial y^2} + \frac{\partial^2 U}{\partial z^2} = 0$.

4) State and prove Euler's theorem for a homogeneous function of two variables.

5) If $x = r\cos\theta$, $y = r\sin\theta$, $z = z$ find $\frac{\partial(x,y,z)}{\partial(r,\theta,z)}$.

IV. Answer **any two** of the following. **(2×5=10)**

1) Obtain the reduction formula for $\int \sin^n x\, dx$ and hence evaluate $\int_0^{\pi/2} \sin^n x\, dx$.

2) Evaluate $\int_0^1 x^4(1-x)^{3/2} dx$.

3) Using Leibnitz's rule of differentiation under integral sign. Evaluate $\int_0^{\pi/2} \frac{\log(1+y\sin^2 x)}{\sin^2 x} dx$.

V. Answer **any three** of the following. **(3×5=15)**

1) Show that the points (0, 4,1) (2, 3, –1) (4, 5, 0) and (2, 6, 2) are the vertices of a square.

2) Find the vector equation of a line passing through the points whose position vectors are $\vec{a}$ and $\vec{b}$. Deduce the Cartesian form.

3) Find the equation of the plane passing through the points (2, 2, 1), (9, 3, 6) and perpendicular to the plane $2x + 6y + 6z = 9$.

4) Show that the lines $\frac{x-1}{2}=\frac{y-2}{3}=\frac{z-3}{4}$ and $\frac{x-4}{5}=\frac{y-5}{6}=\frac{z-6}{7}$ are coplanar and find their point of intersection.

5) Find the shortest distance between the lines $\frac{x-1}{2}=\frac{y-2}{3}=\frac{z-3}{4}$ and $\frac{x-2}{3}=\frac{y-4}{4}=\frac{z-5}{5}$.

VI. Answer **any two** of the following. **(2×5=10)**

1) Find the equation of the sphere which passes through the points (1, 0, 0) (0,1, 0) (0, 0, 1) and (2, –1, 1). Find its centre and radius.

2) Derive the equation of a right circular cone in the standard form $x^2+y^2=z^2\tan^2\alpha$.

3) Find the equation of the right circular cylinder of radius 3 and axis $\frac{x-1}{2}=\frac{y-3}{2}=\frac{z-5}{-1}$.

Q.P. Code – 22138

First Semester B.Sc. Degree Examination, November 2017

(Semester Scheme)

Mathematics

Paper I – MATHEMATICS

Time : 3 Hours] *[Max. Marks : 90*

Instructions to Candidates :

1) *Answer ALL questions.*
2) *Answers should be written completely in English.*

I. Answer any **FIFTEEN** of the following : **(15 × 2 = 30)**

1. Find the truth set of $P(x): x^2 + 5x + 6 = 0$ with the replacement set $R[P(x)] = Z$.

2. Write the Negation of "All rational numbers are real and some are integers".

3. Write all the partitions of the set $S = \{a, b, c\}$.

4. Find the inverse of $f : Q \to Q$ defined by $f(x) = 14x + 20$.

5. Find the n^{th} derivative of $\cos^3 x$.

6. Find the n^{th} derivative of $y = \sin(bx + c)$.

7. If $U = \tan^{-1}\left(\frac{x}{y}\right)$, find $\frac{\partial^2 U}{\partial x \partial y}$.

8. If $u = x^2 + y^2$ where $x = t$, $y = t^2$, find $\frac{du}{dt}$.

9. Define Homogeneous function and give an example.

10. If $u = 3x + 5y$, $v = 4x - 3y$, find the Jacobian of (u, v) w.r.t. (x, y).

11. Evaluate $\int_0^{\pi/4} \tan^5 x \, dx$.

12. Evaluate $\int_0^{\pi/2} \sin^2 x \cos^4 x \, dx$.

13. Find the directions rations and direction cosines of the line joining the points $(4, 3, -5)$ and $(-2, 1, -8)$.

14. Find the projection of AB on the line CD where $A = (3, 4, 5)$, $B = (4, 6, 3)$, $C = (-1, 2, 4)$ and $D = (1, 0, 5)$.

15. Find the angle between the lines whose directions ratios are $(1, -1, 2)$ and $(1, 0, -3)$.

16. Find the angle between the planes $3x - 6y + 2z + 5 = 0$ and $6x - 12y + 4z - 3 = 0$.

17. Find the distance between the parallel planes $6x + 2y - 3z + 4 = 0$ and $12x + 4y - 6z - 27 = 0$.

18. Find the centre and radius of the sphere, $x^2 + y^2 + z^2 - 6x + 8y - 10z + 25 = 0$.

19. Write the equation of right circular cylinder in Cartesian form.

20. Find the equation of right circular cone with vertex at the origin, semivertical angle $30°$ and y-axis as its axis.

II. Answer any **TWO** of the following : **(2 × 5 = 10)**

1. If $p(x)$ and $q(x)$ are any two open sentences then prove that $T[p(x) \to q(x)] = [Tp(x)]' \cup T[q(x)]$.

2. If "$a + b$ is odd and a is even then b is odd". Prove by indirect method.

3. Define inverse function. Prove that if $f : A \to B$ is a bijection, then $(f^{-1})^{-1} = f$.

4. If $f : R \to R$ and $g : R \to R$ are two functions defined by $f(x) = 2x$ and $g(x) = 3x - 2$ then show that $(g \circ f)^{-1} = f^{-1} \circ g^{-1}$.

III. Answer any **THREE** of the following : **(3 × 5 = 15)**

1. Find the nth derivative of $e^{ax}\sin(bx+c)$.

2. If $y=(\sin^{-1}x)^2$ show that $(1-x^2)y_{n+2}-(2n+1)y_{n+1}-n^2y_n=0$.

3. State and prove the Euler's theorem for a homogeneous functions of two variables.

4. If $u=x+y^2/x$ and $v=y^2/x$, find $\frac{\partial(u,v)}{\partial(x,y)}$.

5. If $u=f(r)$, where $r^2=x^2+y^2$, show that $\frac{\partial^2u}{\partial x^2}+\frac{\partial^2u}{\partial y^2}=f''(r)+\frac{1}{r}f'(r)$.

IV. Answer any **TWO** of the following : **(2 × 5 = 10)**

1. Obtain the reduction formula for $\int\sec^n x\,dx$, $\forall\, x\in n$.

2. Evaluate $\int_0^1 x^{3/2}(1-x)^{3/2}\,dx$.

3. Using Leibnitz's rule of differentiation under the integral sign, evaluate $\int_0^1\frac{x^a-1}{\log x}\,dx$ where 'a' is a parameter.

V. Answer any **THREE** of the following : **(3 × 5 = 15)**

1. A line makes angles α, β, γ and δ with four diagonals of a cube, then prove that $\cos^2\alpha+\cos^2\beta+\cos^2\gamma+\cos^2\delta=\frac{4}{3}$.

2. Find the equation of the plane passing through a point and parallel to two given lines. Express it in the Cartesian form.

3. Find the image of the point $(2, 3, -1)$ in the plane $3x-y+2z+12=0$.

4. Show that the lines $\frac{x-1}{2}=\frac{y-2}{3}=\frac{z-3}{4}$ and $\frac{x-4}{5}=\frac{y-5}{6}=\frac{z-6}{7}$ are co-planar, and find their point of intersection.

5. Find the shortest distance between the lines $\frac{x-2}{3}=\frac{y-6}{-2}=\frac{z-5}{-2}$ and $\frac{x-5}{2}=\frac{y-3}{1}=\frac{z+4}{-6}$.

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VI. Answer any **TWO** of the following : **(2 × 5 = 10)**

1. Find the equation of the sphere, which passes through the points (1, 0, 0), (0, 1, 0), (0, 0, 1) and which has its centre on the plane $3x - y + z = 2$.

2. Find the equation of the right circular cone, whose axis is $\frac{x-1}{-1} = \frac{y-2}{3} = \frac{z-3}{3}$ and a generator is $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$.

3. Find the equation of the right circular cylinder of radius 3 and axis $\frac{x-1}{2} = \frac{y-3}{2} = \frac{z-5}{-1}$.

Q.P. Code – 22138

First Semester B.Sc. Degree Examination, October/November 2019

(Non-CBCS Semester Scheme)

Paper I – MATHEMATICS

Time : 3 Hours] *[Max. Marks : 90*

Instructions to Candidates :

1. *Answer ALL Questions.*
2. *Answer should be written completely in English.*

I. Answer any **FIFTEEN** of the following : **(15 × 2 = 30)**

1. Write the truth set of $|x-1|<2$ where the replacement set is the set of all real numbers.

2. Define an open sentence and quantifier.

3. Define equivalence class.

4. Show that $f:Q\to Q$, where Q is the set of all rational numbers, defined by $f(x)=2x+3$ is a bijective mapping.

5. Find the nth derivative of $\cos^2 x$.

6. Find the nth derivative of $\frac{1}{3x-1}$.

7. If $u=\tan^{-1}(y/x)$, find $\frac{\partial^2 u}{\partial x\,\partial y}$.

8. If $u=\sin^{-1}\left(\frac{x}{y}\right)+\tan^{-1}\left(\frac{y}{x}\right)$, then show that $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=0$.

9. If $y^2=4ax$, find $\frac{dy}{dx}$ using partial derivatives.

10. If $x=r\cos\theta$, $y=r\sin\theta$, find $\frac{\partial(x,y)}{\partial(r,\theta)}$.

11. Evaluate $\int_{0}^{\pi/2} \cos^6 x \, dx$.

12. Evaluate $\int_{0}^{\pi/2} \sin^6 x \cos^8 x \, dx$.

13. Find the direction cosines of the line joining the points (2, -3, 6) and (3, -1, -6).

14. Show that the lines whose direction ratios are (2, 3, 4) & (1, -2, 1) are at right angle.

15. Find the equation of the plane passing through (1, 1, 1), (1,-1, 1) and (-7, -3, -5).

16. Find the angle between the planes $3x-6y+2z+5=0$ and $4x-12y+3z-3=0$.

17. Find the angle between the line $\frac{x-3}{2}=\frac{y+1}{-1}=\frac{z+4}{3}$ and the plane $2x+3y-z-4=0$.

18. Find the centre of radius of the sphere $x^2+y^2+z^2-6x+2y-2z-18=0$.

19. Write the equation of the right circular cylinder in Cartesian form.

20. Find the equation of the right circular cone with vertex at origin, semi vertical angle 30^0 and axis along of z-axis.

II. Answer any **TWO** of the following : **(2 × 5 = 10)**

21. If $p(x)$ and $q(x)$ are any two open sets, with the same replacement set, then prove that $T[p(x)\wedge q(x)]=T[p(x)]\cap T[q(x)]$.

22. Prove by direct method : If '*p*' is false and $(q\wedge r)\rightarrow p$ is true, then $\sim q \vee \sim r$ is true.

23. Define inverse of a function. If $f:A\rightarrow B$ is a bijection, then prove that the inverse function $f^{-1}:B\rightarrow A$ is also a bijection.

24. If $f:A\rightarrow B$ and $g:B\rightarrow C$ are two bijective functions, then prove that $(g\circ f)^{-1}$ exists and $(g\circ f)^{-1}=f^{-1}\circ g^{-1}$.

III. Answer any **THREE** of the following : **(3 × 5 = 15)**

25. Find the nth derivative of $e^{ax}\cos(bx+c)$.

26. If $y = e^{m\sin^{-1}x}$, then show that $(1-x^2)y_{n+2} - (2n+1)xy_{n+1} - (n^2+m^2)y_n = 0$.

27. If $u = f(r)$ where $r^2 = x^2 + y^2$, show that $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = f''(r) + \frac{1}{r}f'(r)$.

28. If $u = \cos^{-1}\left(\frac{x+y}{\sqrt{x}+\sqrt{y}}\right)$, then show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \frac{-1}{2}\cot u$.

29. If $x = r\cos\theta$, $y = r\sin\theta$, $z = z$, find $\frac{\partial(x,y,z)}{\partial(r,\theta,z)}$.

IV. Answer any **TWO** of the following : **(2 × 5 = 10)**

30. Obtain the reduction formula for $\int_0^{\pi/2} \cos^n x\,dx$.

31. Evaluate $\int_0^{\pi} x\sin^4 x\cos^6 x\,dx$.

32. Using Leibnitz's rule for differentiation under the integral sign, evaluate $\int_0^1 \frac{x^a - 1}{\log x}dx$, where '$a$' is a parameter.

V. Answer any **THREE** of the following : **(3 × 5 = 15)**

33. Show that the two lines whose direction cosines are given by the equations $3l + 4m + 5n = 0$ and $l^2 + m^2 = n^2$ are parallel.

34. Show that the angle between the diagonals of a cube is $\cos^{-1}\left(\frac{1}{3}\right)$.

35. Find the equation of the plane passing through the points (2, 2, 1), (9, 3, 6) and perpendicular to the plane $2x + 6y + 6z = 9$.

36. Find the image of the point (-3, 0, 1) in the plane $4x-3y+2z=19$.

37. Find the shortest distance between the lines

$$\frac{x-3}{1}=\frac{y-5}{-2}=\frac{z-7}{1} \text{ and } \frac{x+1}{7}=\frac{y+1}{-6}=\frac{z+1}{1}.$$

VI. Answer any **TWO** of the following : **(2 × 5 = 10)**

38. Find the equation of the sphere which passes through (1, 0, 0), (0, 1, 0) and (0, 0, 1) and whose centre lies on the plane $3x-y+z=2$.

39. Find the equation of the right circular cone having its vertex at the origin and the circle $y^2+z^2=25$, $x=4$ as the base circle.

40. Find the equation of the right circular cylinder of radius 2 units whose axis lies along the line $\frac{x-1}{2}=\frac{y+3}{-1}=\frac{z-3}{5}$.

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I Semester B.Sc. Examination, November/December 2016
(CBCS) (Semester Scheme)
MATHEMATICS – I
Paper – 1.1 : Algebra and Calculus – 1

Time : 3 Hours   Max. Marks : 90

Instructions : *1) Answer **all** questions.*
*2) Answer should be written completely in **English**.*

PART – A

I. Answer **any 6** questions : **(6×2=12)**

1) Find the n^{th} derivative of $\sin^2 x$.

2) Define points of inflexion.

3) Find the asymptotes parallel to co-ordinate axes to the curve $\frac{a^2}{x^2}+\frac{b^2}{y^2}=1$.

4) Evaluate $\int_0^{\pi} \cos^6 x\, dx$.

5) If $u = e^{xy}$, find $\frac{\partial^2 u}{\partial x \partial y}$.

6) If $z = f(u, v)$, $u = x^2 - y^2$, $v = 2xy$ then prove that $\frac{\partial z}{\partial x} = 2\left[x\frac{\partial z}{\partial u} + y\frac{\partial z}{\partial v}\right]$.

7) Prove that for a given eigen vector of a matrix, there corresponds only one eigen value.

8) Find the value of 'a' such that the matrix $A = \begin{bmatrix} 2 & -2 & a \\ -2 & -1 & 2 \\ 2 & -2 & 3 \end{bmatrix}$ has rank 2.

PART – B

II. Answer **any 6** questions : **(6×3=18)**

1) Find the n^{th} derivative of $\sin 3x.\cos 2x$.

2) Find polar subtangent and polar subnormal for the curve $r = a e^{\theta \cot \alpha}$.

3) Find the radius of curvature for the curve, $r^n = a^n \sin n\theta$.

4) Evaluate $\int_0^{\pi} x \sin^4 x \, dx$.

5) If $u = \log(x^3 + y^3 + z^3 - 3xyz)$ show that, $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z} = 3$.

6) If $u = 3x + 5y$, $v = 4x - 3y$, find $\frac{\partial(u,v)}{\partial(x,y)}$.

7) Find the rank of the matrix, $\begin{bmatrix} 1 & 2 & 3 & 2 \\ 2 & 3 & 5 & 1 \\ 1 & 3 & 4 & 5 \end{bmatrix}$ using row reduced, echelon form.

8) Find the eigen values and eigen vectors of the matrix $\begin{bmatrix} 5 & -1 \\ 4 & 9 \end{bmatrix}$.

PART – C

III. Answer **any 3** questions : **(3×5=15)**

1) If $y = (\sin^{-1}x)^2$, show that $(1 - x^2)\, y_{n+2} - (2n + 1)\, xy_{n+1} - n^2 y_n = 0$.

2) Prove that, $\tan\phi = r\frac{d\theta}{dr}$, for the curve $r = f(\theta)$.

3) Show that for the curve $r^2\cos 2\theta = a^2$, the length of the perpendicular from the pole to the tangent is $a\sqrt{\cos 2\theta}$.

4) Find the pedal equation of the curve, $x^2 + y^2 = 2ax$.

IV. Answer **any 3** questions : **(3×5=15)**

1) Find $\frac{ds}{d\theta}$ and $\frac{ds}{dr}$ for the curve $r = a(1 - \cos\theta)$.

2) Find the radius of curvature for the pedal equation of the curve $p = f(r)$.

3) Find the evolute for the parabola $y^2 = 4ax$.

4) Obtain the reduction formula for $\int \sec^n x \, dx$.

V. Answer **any 3** questions : **(3×5=15)**

1) State and prove Euler's theorem for a function of two variables.

2) If $\sec u = \frac{x^3 + y^3}{x + y}$, prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 2\cot u$.

3) Expand $e^x \sin y$ using Taylor's theorem at $\left(1, \frac{\pi}{2}\right)$ upto second degree.

4) Find the extremum value of the function $1 + \sin(x^2 + y^2)$.

VI. Answer **any 3** questions : **(3×5=15)**

1) Solve completely the system of equations

$2x - y + 3z = 0$

$3x + 2y + z = 0$

$x - 4y + 5z = 0$.

2) Show that the system of equations

$3x + 4y + 5z = a$

$4x + 5y + 6z = b$

$5x + 6y + 7z = c$ have solutions of $a + c = 2b$.

3) Diagonalize the matrix $\begin{bmatrix} 2 & 4 \\ 0 & 5 \end{bmatrix}$.

4) State and prove Cayley-Hamilton theorem.

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I Semester B.Sc. Examination, Nov./Dec. 2015
(Semester Scheme)
MATHEMATICS – I

Time :3 Hours Max. Marks : 90

Instructions *: 1) Answer **all** questions.*
*2) Answer **should** be written completely in **English**.*

I. Answer **any fifteen** of the following : **(15×2=30)**

1) Write the rules of negating quantified open sentence.

2) If p(x) : $x^2<40$ and q(x) : x is a prime division of 210 and R[p(x)] = R [q(x)] = z then find T[p(x) $\wedge$ q(x)].

3) Write the partition of the set {1, 2, 3, 4} corresponding to an equivalence relation { (1, 1) (2, 2) (3, 3) (4, 4) (1, 2) (2, 1) (3, 4) (4, 3) }.

4) Find the inverse of f : Q $\to$ Q defined by f(x) = 14x + 20.

5) Find the n^{th} derivative of $\cos^2 x$.

6) Find the n^{th} derivative of x^2. logx.

7) If $z = e^{2x} \sin 3y$ find $\frac{\partial^2 z}{\partial x \partial y}$.

8) If $u = x^3 + 4x^2y - 2xy^2 + y^3$ prove that

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 3u$$

9) If $u = x^2 + y^2$ where x = t, $y = t^2$ find $\frac{du}{dt}$.

10) If x = u (1 – v), y = uv find $\frac{\partial(x, y)}{\partial(u, v)}$.

11) Evaluate $\int_0^{\pi} \cos^4 x \, dx$.

P.T.O.

12) Evaluate $\int_{-\pi/2}^{\pi/2} \sin^6 x\, dx$.

13) Find the direction ratios and direction Cosines of the line joining the points (4, 3, – 5) and (–2, 1, – 8).

14) Find the projection of AB on the line CD where A = (1, 2, 3), B = (– 1, 0, 2), C = (1, 4, 2) and D = (2, 0, – 1).

15) Find K such that the lines $\frac{x-1}{2}=\frac{y-2}{2k}=\frac{z+1}{-1}$ and $\frac{x+1}{k}=\frac{y+1}{4}=\frac{z-2}{1}$ are parallel.

16) Find the equation of the plane passing through the point (1, – 2, 4) and perpendicular to the line $\frac{x-1}{2}=\frac{y+3}{1}=\frac{z+1}{-1}$.

17) Find the angle between the line $\frac{x-1}{2}=\frac{y+1}{1}=\frac{z-2}{1}$ and the plane $x - y + z + 1 = 0$.

18) Find the centre and radius of the sphere $x^2 + y^2 + z^2 + 2x - 4y - 6z + 5 = 0$.

19) Find the equation of right circular cone with vertex at the origin, semi-vertical angle 30° and y – axis as its axis.

20) Write the equation of the right circular cylinder in the Cartesian form.

II. Answer **any two** of the following : **(2×5=10)**

1) With usual notations prove that
$T[p(x) \wedge q(x)] = T[p(x)] \cap T[q(x)]$.

2) Prove by direct method, given 'p' as false, $(q \wedge r) \rightarrow p$ is true, show that ~qv~r is true.

3) If f : A → B is a bijection then P.T $f^{-1} : B \rightarrow A$ is also bijection.

4) If f : R → R and g : R → R be defined by $f(x) = x^2$ and $g(x) = (2x - 1)$ verify that $(g_o f)^{-1} = f^{-1}{}_o g^{-1}$.

III. Answer **any three** of the following : (3×5=15)

1) Find the n^{th} derivative of $e^{ax} \cdot \sin(bx + c)$.

2) If $y = a\cos(\log x) + b\sin(\log x)$ show that $x^2 y_{n+2} + (2n+1)xy_{n+1} + (n^2+1)y_n = 0$.

3) If $z = e^{ax+by} f(ax - by)$ show that $b\frac{\partial z}{\partial x} + a\frac{\partial z}{\partial y} = 2abz$

4) State and prove Euler's theorem for a homogeneous function of two variables.

5) If $x = r\sin\theta\cos\phi$, $y = r\sin\theta\sin\phi$,

$z = r\cos\theta$ show that $\frac{\partial(x, y, z)}{\partial(r, \theta, \phi)} = r^2\sin\theta$.

IV. Answer **any two** of the following : (2×5=10)

1) Obtain the reduction formula for $\int \sin^n x\, dx$ where n is a +ve integer.

2) Evaluate $\int_0^1 x^{3/2}(1-x)^{3/2} dx$.

3) Using Leibnitz's rule of differentiation under integral sign evaluate

$\int_0^\infty e^{-x} \frac{\sin \alpha x}{x} dx$.

V. Answer **any three** of the following : (3×5=15)

1) Find the direction ratios of two lines which are connected by the relations $l + m - n = 0$, $mn + 6ln - 12lm = 0$.

2) Find the vector equation of the plane passing through three points and express it in Cartesian form.

3) Find the image of the point (– 3, 0, 1) in the plane $4x - 3y + 2z = 19$.

4) Show that the lines $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and $\frac{x-4}{5} = \frac{y-5}{6} = \frac{z-6}{7}$ are coplanar and find their point of interaction.

5) Find the shortest distance between the lines $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and

$\frac{x-2}{3} = \frac{y-6}{4} = \frac{z-5}{5}$.

VI. Answer **any two** of the following : **(2×5=10)**

1) Find the equation of the sphere which passes through the points (3, 4, 2) (2, 0, 5) and having its centre on the line $x - 2y + z = 0 = 5x - z - 2$.

2) Find the equation of the right circular cone generated by revolving the line $\frac{x}{1} = \frac{y}{2} = \frac{z}{3}$ about the line $\frac{x}{-1} = \frac{y}{1} = \frac{z}{2}$.

3) Find the equation of the right circular cylinder of radius 3 and axis $\frac{x-1}{2} = \frac{y-3}{2} = \frac{z-5}{-1}$.

I Semester B.Sc. Examination, November/December 2016
(Semester Scheme) (Old Scheme)
MATHEMATICS
Paper – I (Repeaters)

Time : 3 Hours Max. Marks : 90

***Instructions** : 1) Answer **all** questions.*
*2) Answer should be written **completely** in **English**.*

I. Answer **any fifteen** of the following . **(15×2=30)**

1) Mention the kinds of quantifiers and write their symbol.

2) Write the negation of "All rational numbers are real and some are integers".

3) Define an equivalence relation.

4) If $f : R \to R$, $g : R \to R$ are defined by $f(x) = 2x + 1$, $g(x) = 5 - 3x$ find g_0f.

5) Find the n^{th} derivative of $\frac{1}{(3x-1)}$.

6) Find the n^{th} derivative of $y = \cos^2 x$.

7) If $U = x^6 + y^6$ show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 6u$.

8) Find the total derivative of $\frac{dz}{dt}$ where $z = x^2 + y^2$ and $x = a\cos t$, $y = b\sin t$.

9) If $x = u^2 - v^2$ and $y = 2uv$ show that $J\frac{(x, y)}{(u, v)} = 4(u^2 + v^2)$.

10) If $U = e^{y/x}$ find $\frac{\partial^2 u}{\partial x \partial y}$.

P.T.O.

11) If $z = x^y$ verify $\frac{\partial^2 z}{\partial x \partial y} = \frac{\partial^2 z}{\partial y \partial x}$.

12) Evaluate $\int_0^{\pi/2} \cos^8 x\,dx$.

13) Find the direction ratios and direction cosines of the line joining the points (4, 3, –5) and (–2, 1, – 8).

14) Show that the lines whose direction ratios are (2, 3, 4) and (1, –2, 1) are at right angles.

15) Write the condition for the lines with direction cosines l_1, m_1, n_1 and l_2, m_2, n_2 to be (i) parallel (ii) perpendicular.

16) Find the angle between the planes, $6x - 3y - 2z - 7 = 0$ and $x + 2y + 2z + 9 = 0$.

17) Find the distance between the parallel planes $2x - y + 3z + 4 = 0$ and $12x - 6y + 18z - 18 = 0$.

18) Find the centre and radius of the sphere $x^2 + y^2 + z^2 + 2x - 4y - 6z + 5 = 0$.

19) Find the equation of right circular cone with vertex at the origin, semi-vertical angle 30° and axis along y-axis.

20) Find the equation of the right circular cylinder of radius 2, and whose axes is $\frac{x-1}{2} = \frac{y+3}{-1} = \frac{z-3}{5}$.

II. Answer **any two** of the following : **(2×5=10)**

1) Show that $T[p(x) \vee q(x)] = T[p(x)] \cup T[q(x)]$.

2) Prove the following statement by indirect proof. "If a + b is odd and a is even then b is odd".

3) If $f : A \to B$ is a bijection then prove that $f^{-1} : B \to A$ is also bijection.

4) If R and R' be two equivalence relation on a nonempty set A, show that $R \cap R'$ is also an equivalence relation.

III. Answer **any three** of the following : **(3×5=15)**

1) Find the n^{th} derivative of $e^{ax}\cos(bx+c)$.

2) If $y = a\cos(\log x) + b\sin(\log x)$ show that $x^2 y_{n+2} + (2n+1)xy_{n+1} + (n^2+1)y_n = 0$.

3) State and prove Euler's theorem for a homogeneous function of two variables.

4) If $U = f(r)$ where $r^2 = x^2 + y^2$ then show that $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = f''(r) + \frac{1}{r} f'(r)$.

5) If $x = r\cos\theta,\ y = r\sin\theta$ verify $\frac{\partial(x,y)}{\partial(r,\theta)} \times \frac{\partial(r,\theta)}{\partial(x,y)} = 1$.

IV. Answer **any two** of the following : **(2×5=10)**

1) Obtain the reduction formula for $\int \sin^n x\,dx$ and hence find $\int_0^{\pi/2} \sin^n x\,dx$.

2) Evaluate $\int_0^1 x^3\sqrt{x - x^2}\,dx$.

3) By applying differentiation under the integral sign, show that

$$\int_0^{\pi/2} \frac{\log(1 + y\sin^2 x)}{\sin^2 x}\,dx = \pi(\sqrt{1+y} - 1).$$

V. Answer **any three** of the following : **(3×5=15)**

1) If α, β, γ are the angles made by a line with the co-ordinates axes.

Prove that (i) $\sin^2\alpha + \sin^2\beta + \sin^2\gamma = 2$

(ii) $1 + \cos 2\alpha + \cos 2\beta + \cos 2\gamma = 0$.

2) Find the angle between the two lines whose direction cosines satisfy the equation $l + m + n = 0$ and $2l + 2m - mn = 0$.

3) Find the equation of the plane which makes the intercepts a, b, c on the co-ordinates axes.

4) Show that the lines $\frac{x-1}{2} = \frac{y+1}{-3} = \frac{z+10}{8}$ and $\frac{x-4}{1} = \frac{y+3}{-4} = \frac{z+1}{7}$ are coplanar and find their point of intersection.

5) Find the shortest distance between the lines.

$$\frac{x-1}{2} = \frac{y+2}{3} = \frac{z+3}{2} \text{ and } \frac{x-2}{2} = \frac{y-8}{2} = \frac{z+1}{1}.$$

VI. Answer **any two** of the following : **(2×5=10)**

1) Find the equation of the sphere passing through the origin and making intercept a, b, c on the co-ordinate axes.

2) Derive the equation of a right circular cone in the standard form

$x^2 + y^2 = z^2 \tan^2\alpha$.

3) Find the equation of the right circular cylinder of radius 3 and axis $\frac{x-1}{2} = \frac{y-3}{2} = \frac{z-5}{-1}$.